Reg No.:_______________ Name:_______________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION(2019 SCHEME), DECEMBER 2019

Course Code: MAT101

Course Name: LINEAR ALGEBRA AND CALCULUS

(2019-Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

1	Determine the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$	(3)
2	If 2 is an eigen value of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, without using its characteristic equation, find the other eigen values.	(3)
3	If $f(x,y) = xe^{-y} + 5y$ find the slope of f (x,y) in the x-direction at (4,0).	(3)
4	Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$, where $z = e^x \sin y + e^y \cos x$	(3)
5	Find the mass of the square lamina with vertices (0,0) (1,0) (1,1) and (0,1) and density function $x^2 y$	(3)
6	Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing to polar coordinates.	(3)
7	Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k}{2k+1}$	(3)
8	Check the convergence of $\sum_{k=1}^{\infty} \frac{1}{k^{k/2}}$	(3)
9	Find the Taylors series for $f(x) = \cos x$ about $x = \frac{\pi}{2}$ up to third degree terms.	(3)
10	Find the Fourier half range sine series of $f(x) = e^x$ in $0 < x < 1$	(3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11 a) Solve the system of equations by Gauss elimination method. (7)

$$x + 2y + 3z = 1$$
$$2x + 3y + 2z = 2$$
$$3x + 3y + 4z = 1$$

b) Find the eigenvalues and eigenvectors of (7)

$$\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

12 a) Find the values of λ and μ for which the system of equations (7)

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

has (i) no solution (ii) a unique solution and (iii) infinite solution

b) Find the matrix of transformation that diagonalize the matrix (7)

$A=\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$. Also write the diagonal matrix.

Module-II

13 a) Let f be a differentiable function of three variables and suppose that (7)
$w = f(x-y, y-z, z-x)$,show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$

b) Locate all relative extrema of $f(x,y) = 4xy - y^4 - x^4$ (7)

14 a) Find the local linear approximation L to the function $f(x,y) = \sqrt{x^2 + y^2}$ at the point P(3,4).Compare the error in approximating f by L at the point Q(3.04,3.98) with the distance PQ. (7)

b) The radius and height of a right circular cone are measured with errors of at most 1% and 4%, respectively. Use differentials to approximate the maximum percentage error in the calculated volume. (7)

Module-III

15 a) Evaluate $\iint_R y\,dx\,dy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. (7)

b) Use double integral to find the area of the region enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$. (7)

16 a) Evaluate $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\,dx\,dy$ by reversing the order of integration (7)

b) Use triple integrals to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$. (7)

Module-IV

17 a) Find the general term of the series $1 + \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} + \frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7} + \ldots\ldots$ and use the ratio test to show that the series converges. (7)

b) Test whether the following series is absolutely convergent or conditionally convergent $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k(k+1)}}$ (7)

18 a) Test the convergence of $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \ldots\ldots\ldots \frac{x^k}{k(k+1)} + \text{--------}$ (7)

b) Test the convergence of the series $\sum_{k=1}^{\infty} \frac{(k+1)!}{4!\,k!\,4^k}$ (7)

Module-V

19 a) Find the Fourier series of periodic function with period 2 which is given below $f(x) = \begin{cases} -x\,; & -1 \le x \le 0 \\ x\,; & 0 \le x \le 1 \end{cases}$. Hence prove that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \text{---} = \frac{\pi^2}{8}$ (7)

b) Find the half range cosine series for $f(x) = \begin{cases} kx & 0 \le x \le L/2 \\ k(L-x) & L/2 \le x \le L \end{cases}$ (7)

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a) Find the Fourier series of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 < x < \pi \end{cases}$ (7)

b) Obtain the Fourier series expansion for $f(x) = x^2, \quad -\pi < x < \pi$. (7)

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Final Scheme/ Answer Key for Valuation

Scheme of evaluation (marks in brackets) and answers of problems/key

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PART A

Answer all questions, each carries 3 marks.

1 $A \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ ....(2) , Rank of $A = 3$......(1) (3)

2 $2+\lambda_2+\lambda_3=11$.......(1) $2\lambda_2\lambda_3=36$.........(1) $\lambda_2=3$ and $\lambda_3=6$...........(1) (3)

3 Slope in the x direction $f_x = e^{-y}$ ....(2) slope at (4,0) =1....(1) (3)

4 $\frac{\partial z}{\partial x} = e^x \sin y - e^y \sin x, \quad \frac{\partial^2 z}{\partial x^2} = e^x \sin y - e^y \cos x$ ....(1) (3)

$\frac{\partial z}{\partial y} = e^x \cos y + e^y \cos x, \quad \frac{\partial^2 z}{\partial y^2} = -e^x \sin y + e^y \cos x$ ....(1)

Conclusion........(1)

5 $Mass = \iint \delta(x,y)\,dx\,dy$ ....(1) $\int_0^1\int_0^1 x^2 y\,dx\,dy$ .... (1) 1/6......(1) (3)

6 $\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{\infty} e^{-r^2} r\,dr\,d\theta$ ....(1) $-\frac{1}{2}\int_0^{\frac{\pi}{2}} \left[e^{-t}\right]_0^{\infty} d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} d\theta$ ....(1) $\frac{\pi}{4}$ ....(1) (3)

7 $\underset{k\to\infty}{Lt}\ U_k = \underset{k\to\infty}{Lt}\ \frac{1}{(2+1/k)} = 1/2 \neq 0$........(2) Divergent ....(1) (3)

8 $\underset{k\to\infty}{Lt}\ (U_k)^{1/k} = \underset{k\to\infty}{Lt}\ \frac{1}{k^{1/2}} = 0 < 1$........(2) Convergent .... (1) (3)

9 $f(x)=\cos x \quad f'(x)=-\sin x, f''(x)=-\cos x, f'''(x)=\sin x \quad f''''(x)=\cos x$ ....(1) (3)

$f(\pi/2)=0, \quad f'(x)=-1 \quad f''(\pi/2)=0 \quad f'''(\pi/2)=1 \quad f''''(\pi/2)=0$ .... (1)

$f(x)=\frac{(x-\pi/2)}{1!}\left(-1\right)+\frac{(x-\pi/2)^3}{3!}+$.........(1)

OR Alternate method

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Formula $b_n=\frac{2n\pi}{1+\pi^2 n^2}\left(1-(-1)^n e\right)$.....(1+1) $f(x)=\sum_{n=1}^{\infty}\frac{2n\pi}{1+\pi^2 n^2}\left(1-(-1)^n e\right)\sin n\pi x$ ......(1) (3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11 a) $[A:B]=\begin{bmatrix}1 & 2 & 3 & 1\\ 2 & 3 & 2 & 2\\ 3 & 3 & 4 & 1\end{bmatrix}$ .... (1) $\begin{bmatrix}1 & 2 & 3 & 1\\ 0 & -1 & -4 & 0\\ 0 & 0 & 7 & -2\end{bmatrix}$ ....(3) (7)

x = - 3/7, y = 8/7, z = - 2/7 ------(1+1+1)

b) $\lambda^3-12\lambda^2+39\lambda-28=0$ ..... (2) $\lambda=1,4,7$ ....(2) eigen vectors $[2\ \ -1\ \ 2]^T$, $[-1\ \ 2\ \ 2]^T$, $[-2\ -2\ \ 1]^T$ ....(3) (7)

12 a) (7)

Augmented Matrix .... (1)Reducing $[A:B]\sim\begin{bmatrix}2 & 3 & 5 & 9\\ 0 & -15 & -39 & -47\\ 0 & 0 & \lambda-5 & \mu-9\end{bmatrix}$ .... (3)No

solution when λ=5 and μ ≠ 9 ....(1)

unique solution when λ ≠ 5 and μ may have any value....... (1)

infinite number of solutions when λ =5 and μ =9 ......(1)

b) characteristic equation $\lambda^3-12\lambda-16=0$ ...... (1) Getting λ= -2, -2, 4 .... (1) (7)

Eigen Vectors [1,0,-1] [1,1,0] [1,1,2] .... (1+1+1)

$P=\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ -1 & 0 & 2\end{bmatrix}$ .... (1) Diagonal matrix $D=\begin{bmatrix}-2 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & 4\end{bmatrix}$ .... (1)

Module-II

13 a) Put $r=x-y, s=y-z, t=z-x$ ...(1) $\frac{\partial w}{\partial x}=\frac{\partial w}{\partial r}-\frac{\partial w}{\partial t}$ .... (2) $\frac{\partial w}{\partial y}=-\frac{\partial w}{\partial r}+\frac{\partial w}{\partial s}$ ......(2) (7)

$\frac{\partial w}{\partial z}=-\frac{\partial w}{\partial s}+\frac{\partial w}{\partial t}$ .... (2)

b) $f_x=4y-4x^3, f_y=4x-4y^3$..........(1) (7)

$f_{xx}=-12x^2,\ \ f_{yy}=-12y^2,\ \ f_{xy}=4$..................(1)

$f_x=f_y=0$. Critical points (0,0), (1,1), (−1, −1)......................(2)

(0,0) saddle point,.......(1) (1,1), (−1, −1) point of maxima........................(1+1)

14 a) $L(x,y) = 5 + \frac{3}{5}(x-3) + \frac{4}{5}(y-4)$ ---------- (3) (7)

$L(Q) = 5.008$ ……. (1) $L(Q) - f(Q) = 0.00019,$ ……(1)

$|PQ| = 0.045$ ……. (1) $Error = \frac{|L(Q)-f(Q)|}{|PQ|} = 0.0042$ ……. (1)

b) $V = \frac{1}{3}\pi r^2 h$..........(2) $\log V = \log\frac{1}{3}\pi + 2\log r + \log h$..........(2) (7)

$\frac{dV}{V}\times 100 = 2\frac{dr}{r}\times 100 + \frac{dh}{h}\times 100$........(2) Ans= 6% .... (1)

Module-III

15 a) Region of integration------(1) $\iint_R y\,dx\,dy = \int_0^4 \int_{\frac{y^2}{4}}^{2\sqrt{y}} y\,dx\,dy$ ... (2) (7)

$\int_0^4 (2y^{\frac{3}{2}} - \frac{y^3}{4})\,dy$............(3) $= \frac{48}{5}$...........(1)

OR

Region of integration------(1) $\iint_R y\,dx\,dy = \int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} y\,dy\,dx$ ... (2)

$\int_0^4 (4x - \frac{x^4}{16})\,dy$............(3) $= \frac{48}{5}$...........(1)

b) Region of integration........(1) (7)

Area $= \iint_R dx\,dy$......(1) $\int_0^4 \int_{x^2/2}^{2x} dy\,dx$......(2) $\int_0^4 \left(2x - \frac{x^2}{2}\right)dx$.....(1) $= \frac{16}{3}$.....(2)

OR

Region of integration........(1)

Area $= \iint_R dx\,dy$......(1) $\int_0^8 \int_{y/2}^{\sqrt{2y}} dx\,dy$......(2) $\int_0^8 \left(\sqrt{2y} - \frac{y}{2}\right)dy$.....(1) $= \frac{16}{3}$.....(2)

16 a) Region of integration .... (1) (7)

$\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\,dx\,dy = \int_0^1 \int_0^{2x} e^{x^2}\,dx\,dy$ ....(2) $\int_0^1 e^{x^2}\,2x\,dx$ ......(2) $e-1$...... (2)

b) $V = \iiint_G dV$ ... (1) $= \iint_R \int_1^{5-x} dz\,dy\,dx$.....(2) (7)

$\int_0^{2\pi}\int_0^3 (4 - r\cos\theta)\,r\,dr\,d\theta$ .... (2) $\int_0^{2\pi}(18 - 9\cos\theta)\,d\theta$ ... (1) $=$ 36π..............(1)

OR

$V = \iiint_G dV$ ... (1) $= \iint_R \int_1^{5-x} dz\,dy\,dx$.....(2)

$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}(4-x)\,dy\,dx$ ……….. (2) = 36π…………..(2)

Module-IV

17 a) (7)

$u_k = \dfrac{1.2.3......k}{1\cdot3\cdot5\cdot....(2k-1)}$ ------ (2)

$u_{k+1} = \dfrac{(k+1)!}{1\cdot3\cdot5\cdot....(2k-1)(2k+1)}$ -----(1) $\rho = \lim_{k\to\infty}\dfrac{k+1}{2k+1} = \dfrac{1}{2} < 1$ ---(3)

Hence converges------(1)

b) (7)

$|U_k| = \left|\dfrac{1}{\sqrt{k(1+k)}}\right|$.......(1) $\sum_{k=1}^{\infty} V_k = \sum_{k=1}^{\infty}\dfrac{1}{k}$ *Divergent*........(1) $\underset{k\to\infty}{Lt}\dfrac{U_k}{V_k} = 1$.....(1)

Not Absolute Convergent ---(1)

U1>U2>……(1) $\underset{k\to\infty}{Lt}\ U_k = 0$ ---(1) conditionally convergent---(1)

18 a) (7)

$U_{k+1} = \dfrac{x^{k+1}}{(k+1)(k+2)}$.....(1) $\underset{k\to\infty}{Lt}\dfrac{U_{k+1}}{U_k} = x$ ....(1)

x<1 Convergent, x > 1 Divergent x = 1 test fails ……… (1)

If x=1 $U_k = \dfrac{1}{(k)(k+1)}$.......(1) $\sum_{k=1}^{\infty} V_k = \sum_{k=1}^{\infty}\dfrac{1}{k^2}$ *convergent*......(1) $\underset{k\to\infty}{Lt}\dfrac{U_k}{V_k} = 1 \neq 0$.......(1)

Convergent ------(1)

OR

$\underset{k\to\infty}{Lt}\left|\dfrac{U_{k+1}}{U_k}\right| = |x|$ ....(1) $-1 < x < 1$ series convergences.........(1)

x = -1, *U_k decreases & lim $U_k = 0$, so it converge at $x = -1$*---------- (1)

If x=1 $U_k = \dfrac{1}{(k)(k+1)}$.......(1) $\sum_{k=1}^{\infty} V_k = \sum_{k=1}^{\infty}\dfrac{1}{k^2}$ *convergent*......(1) $\underset{k\to\infty}{Lt}\dfrac{U_k}{V_k} = 1 \neq 0$.......(1)

Convergent ------(1)

b) $U_{k+1} = \dfrac{(k+2)!}{4!(k+1)!4^{k+1}}$.......(2) $\underset{k\to\infty}{Lt}\dfrac{U_{k+1}}{U_k} = 1/4 < 1$......(4)Convergent……. (1) (7)

Module-V

19 a) (If the answer is correct without writing the formula give full mark) (7)

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos n\pi x+b_n \sin n\pi x \text{ ....(1)}$$

$Formula\ a_0=2\int_0^1 x\,dx=1$....(1+1) $Formula\ a_n=2\int_0^1 x\cos n\pi x\,dx=\frac{2\left((-1)^n-1\right)}{n^2\pi^2}$.......(1+1)

$b_n=0$........(1)

Deduction----(1)

OR

$$f(x)=a_0+\sum_{n=1}^{\infty} a_n \cos n\pi x+b_n \sin n\pi x \text{ .... (1)}$$

$Formula\ a_0=\int_0^1 x\,dx=\frac{1}{2}$....(1+1) $Formula\ a_n=2\int_0^1 x\cos n\pi x\,dx=\frac{2\left((-1)^n-1\right)}{n^2\pi^2}$.......(1+1)

$b_n=0$........(1)

Deduction----(1)

b) (If the answer is correct without writing the formula give full mark.) (7)

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} \text{ ....(2)}$$

$Formula, a_0=\frac{2}{L}\int_0^L f(x)\,dx=\frac{kL}{2}$...(1+1)

$Formula\ a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx=\frac{2kL}{n^2\pi^2}\left(2\cos\frac{n\pi}{2}-1-(-1)^n\right)$......(1+2)

OR

$$f(x)=a_0+\sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} \text{ ....(2)}$$

$Formula, a_0=\frac{1}{L}\int_0^L f(x)\,dx=\frac{kL}{4}$...(1+1)

$Formula\ a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx=\frac{2kL}{n^2\pi^2}\left(2\cos\frac{n\pi}{2}-1-(-1)^n\right)$......(1+2)

20 a) (If the answer is correct without writing the formula give full mark) (7)

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx \text{ ....(1)}$$

$$Formula, a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx=\frac{1}{\pi}\int_{0}^{\pi} x^2 dx=\frac{\pi^2}{3} \text{ ....(1+1)}$$

$$Formula, a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx=\frac{1}{\pi}\int_{0}^{\pi} x^2 \cos nx dx=\frac{2(-1)^n}{n^2} \text{ ....(1+1)}$$

$$Formula, b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx=\frac{1}{\pi}\int_{0}^{\pi} x^2 \sin nx dx=\frac{2((-1)^n-1)}{\pi n^3}+\frac{\pi(-1)^{n+1}}{n}$$

....(1+1)

OR

$$f(x)=a_0+\sum_{n=1}^{\infty} a_n \cos n\pi x + b_n \sin n\pi x \text{ .... (1)}$$

$$Formula, a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx=\frac{1}{2\pi}\int_{0}^{\pi} x^2 dx=\frac{\pi^2}{6} \text{ ....(1+1)}$$

$$Formula, a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx=\frac{1}{\pi}\int_{0}^{\pi} x^2 \cos nx dx=\frac{2(-1)^n}{n^2} \text{ ....(1+1)}$$

$$Formula, b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx=\frac{1}{\pi}\int_{0}^{\pi} x^2 \sin nx dx=\frac{2((-1)^n-1)}{\pi n^3}+\frac{\pi(-1)^{n+1}}{n}$$

....

(7)

b)

(If the answer is correct without writing the formula give full mark)

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx \text{ ....(2)}$$

$$Formula, a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx = \frac{2\pi^2}{3} \ldots\ldots\ldots\ldots(1+1)$$

$$Formula, a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx = \frac{4(-1)^n}{n^2} \ldots\ldots\ldots\ldots(1+1)$$

$$Formula,\ b_n = 0 \ldots\ldots\ldots\ldots(1)$$

OR

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\,x + b_n \sin n\,x \ldots\ldots\ldots\ (2)$$

$$Formula, a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)dx = \frac{\pi^2}{3} \ldots\ldots\ldots\ldots(1+1)$$

$$Formula, a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx dx = \frac{4(-1)^n}{n^2} \ldots\ldots\ldots\ldots(1+1)$$

$$Formula,\ b_n = 0 \ldots\ldots\ldots\ldots(1)$$

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